# Damage Identification of Gravity Dams using Free Vibration Data

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**Abstract**

Damage detection of important structures based on nondestructive techniques and response-based methods have widely been one of the most interesting fields of engineering for the last years. Besides, it is clear that taking fluid-structure interaction into account impact dynamic responses of dams and other fluid-structure systems. As a result, it is desired to implement the damage detection process of structures in contact with water regarding fluid-structure interaction. In fact, under these circumstances, the results of damage detection problem would be more reliable since the modeling errors decline. On this basis, in the present work, an ideal gravity dam with a fully-filled reservoir is discussed to identify the location and extent of its damage, employing free vibration data extracted from the finite element model of the dam.

**Keywords: damage detection, incomplete modal data, model updating, fluid-structure interaction, gravity dam.**

## 1. INTRODUCTION

Engineering structures are exposed to unexpected loads and undesirable issues such as corrosion that may cause damages. Consequently, the structural health monitoring and nondestructive damage detection can play an important role to identify structural damages by assessing the changes in responses of damaged structures. To this end, there has been a wide range of studies into presenting various damage identification methods for different structures. It should be noted that the vast majority of this method have been proposed for structure-only systems [1]. On the other hand, the fluid-structure systems have been increasingly applied in some field of engineering including oil and gas industries. As a result, investigation of damage detection problem in fluid-structure interaction systems can be an interesting and prominent field of research.

Model-based damage detection methods are recognized as one of the efficient approaches to identify the extent and location of structural damages [2, 3]. In fact, by comparison between the responses of damaged and undamaged phases of a system and extraction a set of equations, the damage detection problem is implemented in this kind of methods. Therefore, the data of the undamaged and damaged phases of a system including vibration data, time history responses and static displacements are required to execute the damage detection process. Vibration measurements have been extensively used to establish the damage detection equations due to its high sensitivity to damages.That is to say thatthe vibration data can provide local information about existing damages [4].

Apart from proposed methods for systems in contact with air, some researchers have been studied the damage detection problem of systems in contact with liquid [5, 6]. It should be noted that in most of these studies, the influence of fluid on the damage detection formulation has been ignored, and the fluid-structure systems have been considered as structure-only systems. In another work, Sotoudehnia et al.[1] presented a new set of damage detection equations for systems in contact with water considering therigorous fluid-structure interaction. They located damages in a two-dimensional concrete tank and showed the performance of their method in both structure-only and fluid-structure systems. The frequency and mode shape of the first coupled mode were employed, and the fluid pressures were expressed based on the structural part of the used mode shapes [1].

As mentioned before, model-based methods lead to a set of damage detection equations either linear or nonlinear, overdetermined or underdetermined. There are different methods to solve the system of equations ranged from optimization to algebraic methods depending on its type [7].To solve a linear set of equations, bothdirect and iterative methods can be utilized regarding the characteristics of the equations. In the literature, the least square method,as a direct method, has been widely applied, and its performance has been demonstrated. Besides, Sotoudehnia et al. [8] deployed biconjugate gradient method, as an iterative method in order to solve the damage detection equations extracted from time history analysis.

In what follows, a brief review of the free vibration of fluid-structure systems is conducted.The eigenvalue problem of fluid-structure systems is coupled, asymmetric and linear, and its solution can be provided by: (1) basic methods, which compute modal data considering the coupled non-symmetric eigenvalue problem [9] and (2) approaches that tend to switch the coupled problem to decoupled one. In this paper, the former methods are used to solve the eigenvalue problem [10].

In the current paper, the identification of location and extent of damages in an ideal gravity dam is discussed using the coupled eigenvalue problem governing the free vibration of fluid-structure systems. The set of damage detection equations areestablished based on incomplete modal data in which the solid displacements are considered as the measured DOFs, and the fluid pressures are perceived as the unmeasured DOFs. Three different scenarios, which consist of minor and major damages separately, are defined to simulate the model of the damaged system since the experimental test has not been conducted. To model the damage elements, a reduction in Young's modulus of the corresponding elements is induced. In the end, a numerical study is carried out to assess the precision of the proposed method. The results prove the efficiency of the method to identify the place and extent of damages in gravity dams.

## 2. GOVERNING EQUATIONS

The finite element eigenvalue problem of gravity dams considering fluid-structure interaction for the $i^{th}$ mode can be expressed as follows [10]:

$$\left(\begin{bmatrix}\mathbf{K} & -\mathbf{B}^{\mathrm{T}}\\ 0 & \mathbf{H}\end{bmatrix}-\omega_i^2\begin{bmatrix}\mathbf{M} & 0\\ \mathbf{B} & \mathbf{G}\end{bmatrix}\right)\begin{Bmatrix}\mathbf{r}_i\\ \mathbf{p}_i\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix} \quad (1)$$

Herein, mass and stiffness matrices of the structural part are shown by $\mathbf{K}$ and $\mathbf{M}$ of size $ns \times ns$, respectively. $ns$ isdenoted to the number of structural degrees of freedom (DOFs). Furthermore, $\mathbf{H}$ and $\mathbf{G}$ arethe characteristics matrices of the fluid domain of size $nf \times nf$. Moreover, $nf$ is the number of fluid DOFs. The interaction matrix is presented by matrix $\mathbf{B}$ of size $nf \times ns$. Besides, the $i^{th}$ frequency of the system is presented by $\omega_i$ and its correspondingnodal displacements of the structural part and nodal pressures of the fluid domainin the $i^{th}$ mode shape (eigenvector) are demonstrated by $\mathbf{r}_i$ and $\mathbf{p}_i$, correspondingly.

The characteristic matrices of the individual fluid elements and the interaction matrix can be computed as below:

$$\mathbf{H}_e=\iint_e\left(\left(\frac{\partial \mathbf{N}_f}{\partial x}\right)^{\mathrm{T}}\frac{\partial \mathbf{N}_f}{\partial x}+\left(\frac{\partial \mathbf{N}_f}{\partial y}\right)^{\mathrm{T}}\frac{\partial \mathbf{N}_f}{\partial y}\right)dxdy \quad (2)$$

$$\mathbf{G}_e=\frac{1}{c^2}\iint_e \mathbf{N}_f^{\mathrm{T}}\mathbf{N}_f\,dxdy \quad (3)$$

$$\mathbf{B}_e=\int_B \mathbf{N}_f^{\mathrm{T}}\mathbf{N}_s\,dB \quad (4)$$

Herein, $\mathbf{N}_f$ and $\mathbf{N}_s$ are interpolation functionrelated to the fluid domainand the structural part, correspondingly. Moreover, $c$ is wave propagation speed in the fluid domain. Finally, the global matrices ($\mathbf{H}$, $\mathbf{G}$ and $\mathbf{B}$) can be calculated by assembling the individual matrices.

## 3. DAMAGE DETECTION USING MODAL DATA

In this section, the damage detection equations are established using the presented method in reference [1]. In the following, the mentioned method is briefly discussed.To implement the damage detection problem, the complete modal data are required to present a mathematical relation between the damaged structure and the finite element model, whereas, the modal data of damaged system are incomplete. As mentioned above, a mode shape of the fluid-structure system consists of solid displacements and fluid pressures. In our work, it is presumed that only the solid DOFs of the damaged structure are measured and the fluid ones are unmeasured due to the practical problems in measuring fluid pressures. To tackle this difficulty, the fluid DOFs are expressed based on the solid ones through expanding the second set of Eq. (1). It yields [1]:

$$\mathbf{p}_i = (\mathbf{H} - \omega^2 \mathbf{G})^{-1} \omega^2 \mathbf{B} \mathbf{r}_i \quad (5)$$

Expanding the first set of Eq. (1) and substituting Eq. (5) into itleads to:

$$\mathbf{K}\mathbf{r}_i - \mathbf{B}^{\mathrm{T}} (\mathbf{H} - \omega_i^2 \mathbf{G})^{-1} \omega_i^2 \mathbf{B}\mathbf{r}_i - \omega_i^2 \mathbf{M}\mathbf{r}_i = 0 \quad (6)$$

Note that; Eq. (6) is based on the solid displacements of the $i^{th}$ mode shape.Then, using this relation, the damage detection equations are constructed. At first, it is required to simulate the damaged system utilizing the finite element model since the experimental data are not available. For this purpose, damages are considered as a reduction in Young's modulus of the finite element model. That is to say, the stiffness matrix of the damaged system is different to the undamaged one while the mass matrices of two phases (damaged and undamaged) are the same. Accordingly, Eq. (6) is rewritten for the damaged phase as follows:

$$\mathbf{K}_d \mathbf{r}_{id} - \mathbf{B}^{\mathrm{T}} (\mathbf{H} - \omega_{id}^2 \mathbf{G})^{-1} \omega_{id}^2 \mathbf{B}\mathbf{r}_{id} - \omega_{id}^2 \mathbf{M}\mathbf{r}_{id} = 0 \quad (7)$$

It should be noted that subscript $d$ refers to the characteristics of the damaged system.The stiffness of damaged system can be expressed regarding the stiffness of undamaged structure and changes in stiffnesses of individual elements owing to existing damages as:

$$\mathbf{K}_d = \mathbf{K} + \sum_{j=1}^{ne} \alpha_j \mathbf{K}_j^e \quad (8)$$

where the stiffness matrix of the undamaged phase and the damage extent for the $j^{th}$ individual element are shown by $\mathbf{K}_j^e$ and $\alpha_j$, respectively. Then, substituting Eq. (8) into Eq. (7) yields:

$$\mathbf{K}\mathbf{r}_{id} + \sum_{j=1}^{ne} \alpha_j \mathbf{K}_j^e \mathbf{r}_{id} - \mathbf{B}^{\mathrm{T}} (\mathbf{H} - \omega_{id}^2 \mathbf{G})^{-1} \omega_{id}^2 \mathbf{B}\mathbf{r}_{id} - \omega_{id}^2 \mathbf{M}\mathbf{r}_{id} = 0 \quad (9)$$

$$\sum_{j=1}^{ne} \alpha_j \mathbf{K}_j^e \mathbf{r}_{id} = \underbrace{(\mathbf{B}^{\mathrm{T}} (\mathbf{H} - \omega_{id}^2 \mathbf{G})^{-1} \omega_{id}^2 \mathbf{B} + \omega_{id}^2 \mathbf{M} - \mathbf{K})}_{\mathbf{A}_i} . \mathbf{r}_{id} \quad (10)$$

$$\mathbf{r}_{id} = (\mathbf{A}_i)^{-1} \sum_{j=1}^{ne} \alpha_j \mathbf{K}_j^e \mathbf{r}_{id} \quad (11)$$

Finally, after some mathematical operations, the damage detection equations considering the fluid-structure interaction arestablished with coming appearance [1]:

$$\begin{Bmatrix} \mathbf{r}_{1d} \\ \vdots \\ \mathbf{r}_{id} \\ \vdots \\ \mathbf{r}_{md} \end{Bmatrix} = \begin{bmatrix} \mathbf{S}_1 \\ \vdots \\ \mathbf{S}_i \\ \vdots \\ \mathbf{S}_m \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{ne} \end{Bmatrix} \quad (12)$$

where the number of measured modes is equal to $m$. Furthermore, $\mathbf{S}_i$ is the coefficient matrix whose $j^{th}$ column is computed using $\mathbf{r}_{id}$ and the stiffness matrix of the $j^{th}$ element. It means:

$$\mathbf{S}_i(:,j) = (\mathbf{A}_i)^{-1}\mathbf{K}_j^e\mathbf{r}_{id} \tag{13}$$

It worth mentioning that the mentioned equations of damage detection problem are overdetermined to estimate the location and amount of damage more accurately [1].Furthermore,least-square method is applied to solve the set of damage detectionequations.

## 4. NUMERICAL STUDY

A finite element model of adamaged gravity dam with a fully-filled reservoir based on plane strain formulation, as shown in Fig. 1, is employed to validate the proposed method to identify the place and extent of damage. As aforesaid, damaged elements in the model are induced by a reduction in Young's modulus. The dam is 200 $\mathrm{m}$ high and 160 $\mathrm{m}$ wide, and the ratio of increase in the width to height is 5 to 4. Moreover, its physical properties are $E = 2.1\times10^5$ MPa (Young's modulus);and $\rho_s = 2400$ kg/m$^3$ (density).The wateris considered as a compressible, inviscid and irrotational fluid with density 1000 kg/m$^3$ and wave propagation speed 1440 m/s . It is worthwhile to mention that eight-node solid and fluid elements are employed to model the gravity dam with two translational DOFs for the solid nodes and one pressure DOF for the fluid ones.

Two various lengths of the fluid domain ( $L = H,\ 2H$ ) are selected to investigate the influence of the number of fluid DOFs on damage detection results. Note that; in case $L = H$ , the number of fluid DOFs is equal to 85, and its counterpart in case $L = 2H$ is identical to 160. Besides, the number of solid DOFs in both cases is 124.

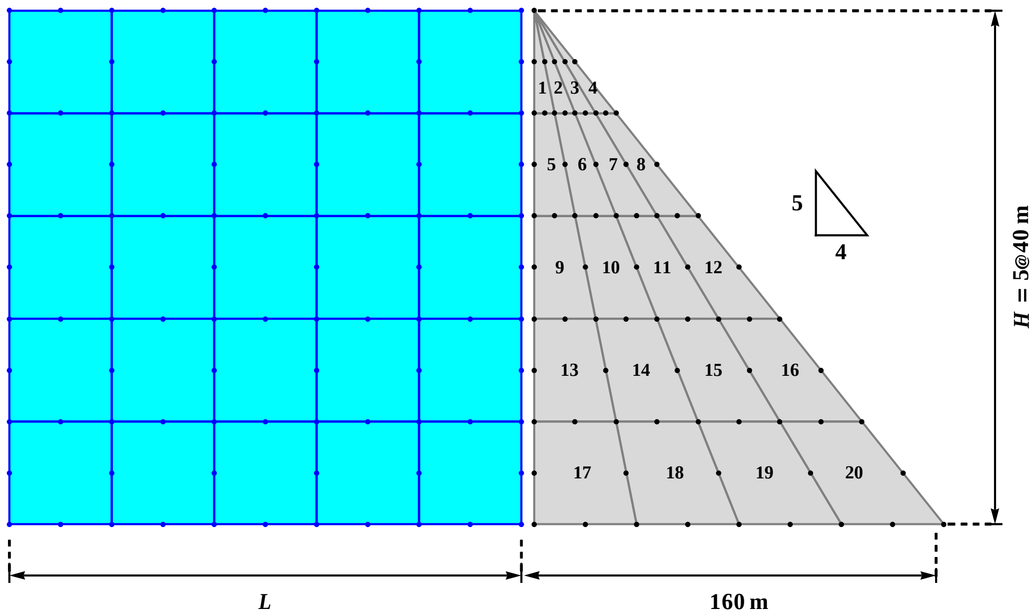


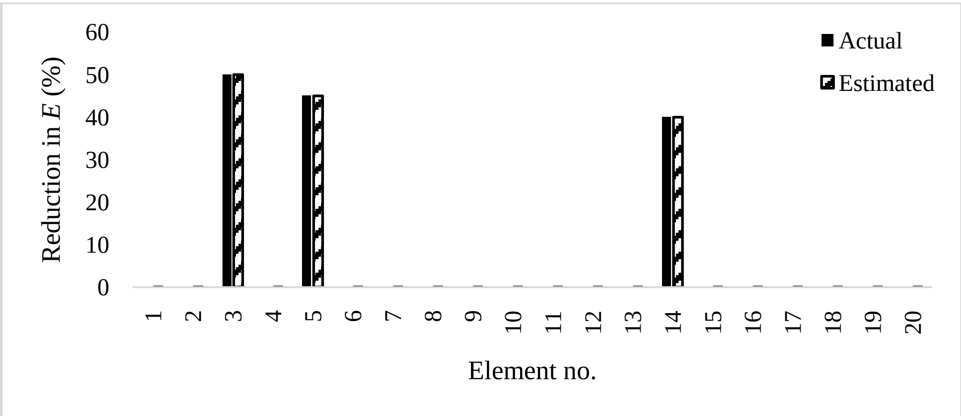

**Figure 1. Finite element model of the gravity dam-reservoir**

**Table 1- Two selected damage scenarios**

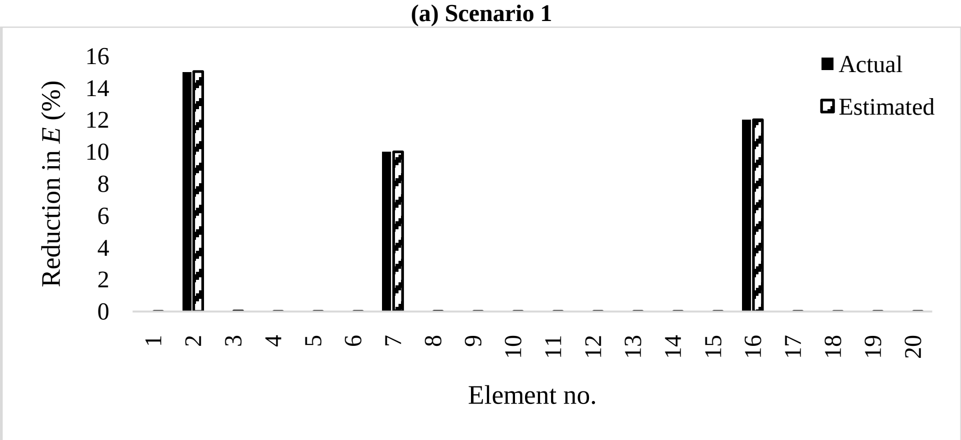
| Scenario no. | Element no. | Reduction in Young's modulus (%) |
|---|---|---|
| 1 | 3 | 50 |
| | 5 | 45 |
| | 14 | 40 |
| 2 | 2 | 15 |
| | 7 | 10 |
| | 16 | 12 |
| 3 | 1 | 20 |
| | 5 | 20 |
| | 9 | 20 |
| | 13 | 20 |
| | 17 | 20 |

Three damage scenarios are defined to model the damaged phase of the system regarding element numbers depicted in Fig. 1, as tabulated in Table 1. The first scenario includes major damages, and the extent of damages in the second one are limited to 15 %. Moreover, in scenario 3, it is assumed that all of the elements in contact with water (wetted solid element) are damaged

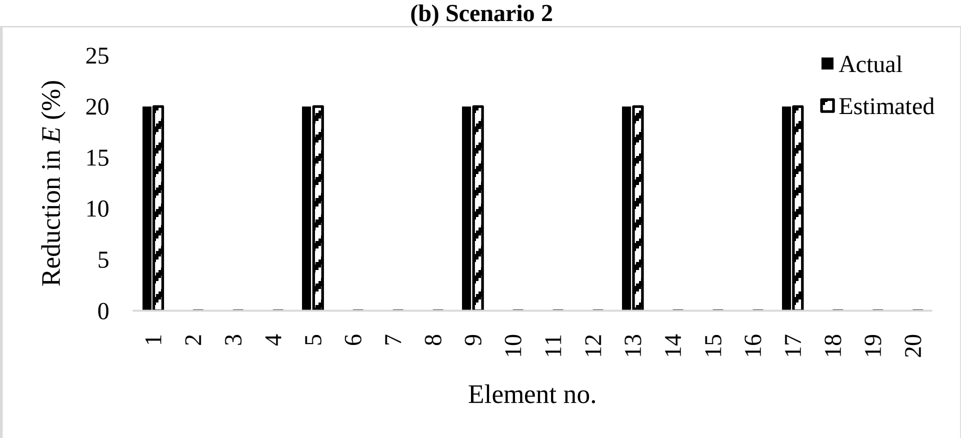
The results of damage detection problem for the gravity dam based on both values of $L$ and three defined scenarios are shown in Figs. 2 and 3.The valuable point is that the modal data of the first mode has been only employed. Consequently, the number of equations is equal to 124 since the number of measured DOFs is 124.Moreover, the number of unknowns (the number of structural elements) is 20.




**(a) Scenario 1**




**(b) Scenario 2**

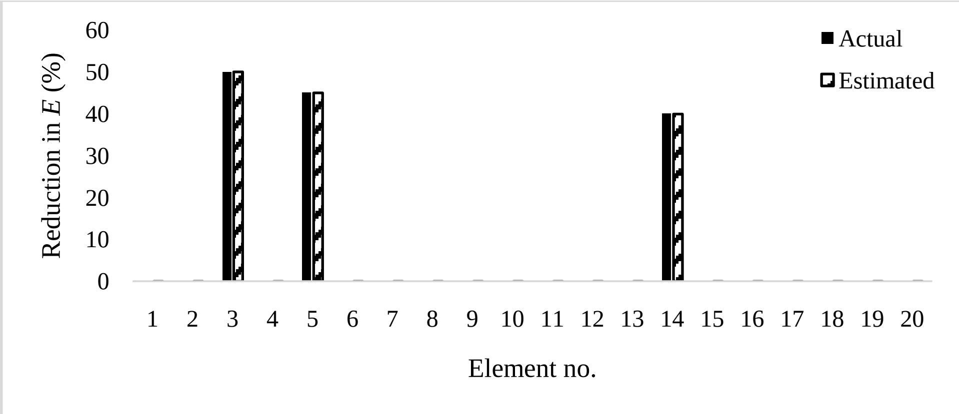



**(c) Scenario 3**

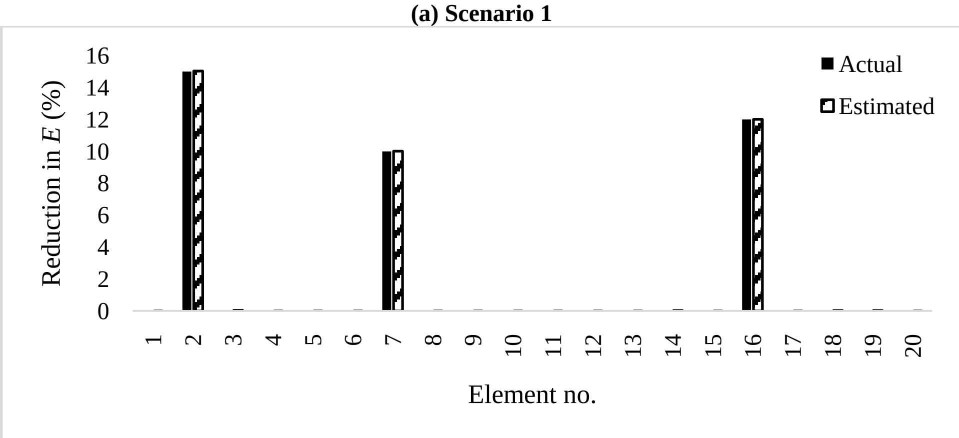
**Fig. 2. Damage detection results for case *L=H***

Findings prove the efficiency and high accuracy of the proposed method to identify the location and extent of damages in the dam. As it can be seen from these figures, the damage localization and damage quantification have been carried out with desired results. Moreover, the capability of the proposed method for damage identification of the gravity dam concerning minor and major damages has been demonstrated.

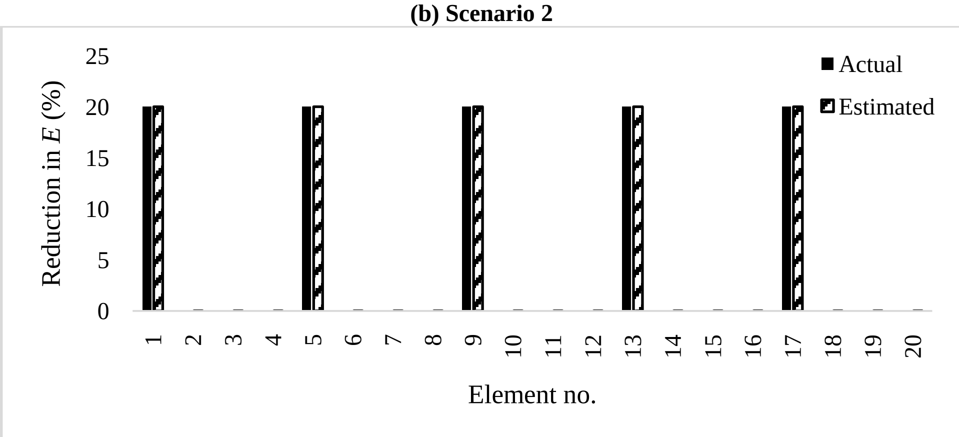
Another observation in these figures is that the number of unmeasured fluid pressures does not exert a considerable impact on the results of the damage detection process. Consequently, the expression of fluid DOFs based on measured solid DOFs to overcome the difficulty of incomplete modal data could be useful. In other words, the results of damage detection problem of the gravity dam for *L/H*=1 and 2 are the same, whereas, the number of unmeasured fluid pressures in case *L/H*=1 is less than case L/H=2 (85 VS. 160).




**(a) Scenario 1**




**(b) Scenario 2**




**(c) Scenario 3**

**Fig. 3. Damage detection results for case *L*=2*H***

## 5. CONCLUSIONS

This paper has aimed to identify the amount and place of damages in gravity dams considering the fluid-structure interaction.In fact, theproposed damage detection method has considered the interaction effect using the coupled eigenvalue problem of the fluid-structure systems to extract the damage detection equations.Three scenarios have been defined to model the damaged system in which the damages are considered as a reduction in Young's modulus. Findings proved the capability of the proposed method for damage detection of gravity dams. It has been shown that the number of fluid DOFs, which are considered as unmeasured DOFs, does not have considerable effect on the results of damage detection problem.

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