Importance of Multiple Time Step Optimization in River Basin Planning and Management -- the Case Study of Narmada and Damodar River Basin

Nesa Ilich[1], Ashoke Basistha[2]

Abstract

River basin planning studies are conducted to help define the best water management that matches increasing demands and limited water supply. River basin models have become an essential tool for planning studies, allowing various management policies to be evaluated by developing and analyzing various river basin modelling scenarios. Most river basin management agencies use operating rules which were developed by simulation models that relied on the use of reservoir rule curves. The rule curves are the guidelines for reservoir operators which suggest preferable water levels for each point in time within the calendar year. Most rule curves do not change from year to year, and they typically assume the same starting water level at the beginning of the hydrologic year. This paper outlines the importance of using multiple time step optimization (MTO), which can provide simultaneous optimization of water supply as well as optimal amount of demand hedging. The results of MTO based simulation can be used in various ways to evaluate the validity and usefulness of the existing rule curves, as we as to help generate dynamically adjustable operating rules that take into account variable starting storage levels and seasonal flow forecasts when available. This paper presents the MTO results of the Damodar River Basin, which shows a potential increase in generated hydro power, while simultaneously eliminating flood damage in all simulated years without any detrimental effects on water supply to irrigation and municipalities. The paper also contains some guidelines on how the results of planning studies can be implemented to improve future river basin management for seasonal operation.

---

[1] Senior River Basin Modelling Consultant, NHP Project, Ministry of Water Resources, New Delhi, India. Corresponding author. Email: nilich@optimal-solutions-ltd.com

[2] Senior Hydrologist and Hydrological Modelling Expert, NHP Project, Ministry of Water Resources, New Delhi, India

INTRODUCTION

Use of computer models has become an integral part of river basin planning and management. There is a large number of computer models used in the water resources sector, which differ in many ways depending on their ultimate goals. This paper is focused on river basin management models, which are designed to mimic decision making that river basin managers are faced with on a daily basis -- finding the most suitable set of reservoir releases and water use over a specified time horizon. Several modelling approaches have been used in the past, being generally divided into "rule based" and "optimization based", where optimization models can further be subdivided into several categories, depending on the nature of the optimization engine (e.g. heuristic solvers or solvers based on some form of mathematical programming). Regardless of the actual modeling approach, most model vendors claim outstanding fitness of their models for addressing river basin management tasks. This is primarily the result of the lack of stringent acceptance criteria among water resources practitioners. In this context, the meaning of the word "optimization" has been diluted, since everyone seems to claim that their models possess ability to "optimize" the search for the best solution. Recent initiatives related to the model selection process for Narmada River Basin in India were aimed at introducing more technical vigour into this field, thus setting higher expectations for model vendors regarding future applications of their models (Ilich et al, 2019).

Decision making process in water resources models are usually based on either a set of user specified "what-if" rules, or they use some type of mathematical optimization algorithm which is aimed to find the best set of hypothetical regulated flows that minimizes or maximizes a given objective function. These models differ from descriptive type of models as they simulate decisions of reservoir operators and river basin managers. For example, at times of water shortage, these models should have capabilities to provide solutions that can bypass upstream users and provide water to downstream users that have higher priorities. To demonstrate a wide variety of the existing models, literature review of reservoir operation models for basin planning purposes was compiled by Wurbs (1993), and subsequently updated by Labadie (2004). These two papers contain a short review of more than 50 models. In spite of this, when it comes to water management models, there are no clear favourites among the practitioners, unlike other specific areas such as river hydraulics, where the undisputed and universally accepted models such as for example the HEC-RAS (Hydrologic Engineering Centre, 2006) or Mike 11 (Danish Hydraulic Institute, 2019) are generally deemed to be the top in their class. While some model vendors have invested considerable effort in advertising and promotion of their models, the reputation of model capabilities and its performance can only be demonstrated by providing successful solutions to the test problems, which is sorely lacking in technical literature. A list of universally acceptable test problems has yet to be established among the practitioners.

Research efforts in academia have been disproportionally directed towards investigation of the development and use of heuristic solvers. Most heuristic solvers are based on simulation of some kind of biological process found in nature, and they are generally referred to as "evolutionary algorithms" (Maier et al, 2014). These algorithms are interesting from the research standpoint, but their abilities to solve large problems with double precision variables (i.e. find target diversion flows and reservoir levels) lags far behind the well-established Linear Programming, which has been the foundational solution technique behind most successful models among the practitioners.

The first optimization-based river basin management models for water distribution along water resources networks used simple linear programming algorithms specifically developed for the optimization of network flow problems, generally known as Network Flow Algorithms (NFA). While the NFA solvers offer high solution speed, they cannot easily represent dynamic constraints that exist in water resources networks. Their problem domain representation was limited to the mass balance at each node, and fixed upper and lower bounds on flow in each channel or river reach. Constraints such as the return flows from irrigation districts that depend on allocated consumptive use, or maximum reservoir outflows that are a function of average storage over a time step were initially addressed within NFA solvers using an iterative approach. However, iterations often fail to find optimal solutions, and in some instances actually take the solution process in the wrong direction (Ilich, 2008). In spite of this, many NFA based models such as MODSIM (Colorado State University, 2020), REALM (Gov. of Victoria, 2020), HEC-ResPRM

(US Corps of Engineers, 2020) or AQUATOOL (Haro et al, 2012) became popular among practitioners. Nevertheless, the shortcomings of the NFA models were noted, and this led to the development of models based on commercial mixed integer solvers applied in models such as OASIS (Rundall et al, 1998), RIVERWARE (Zagona et al, 2001), HEC-FCLP (Needham et al, 2000), and WEB.BM (Ilich, 2019). These models use some form of Mixed Integer Programming (MIP) solver, since binary variables were required to ensure that reservoir zones fill from bottom to top, and empty from top to bottom (Ilich, 2008).

THE CONCEPT OF RESERVOIR RULE CURVES

The principal disadvantage of solving individual time steps is displayed in the two graphs at the bottom of Figure 1 below, which show crop failure in both years. To avoid it, irrigation managers typically hedge their demands in extremely dry years – in other words, they lower their targets to reduce the chances of crop failure. Their dilemma is then to determine the level of reduction that is most appropriate for the current conditions.

Historically, Ravelle (1974) developed the concept of the reservoir rule curve in an effort to avoid the STO solutions shown as a dashed line in the top left diagram in Figure 1. This allows model users to input a maximum permissible drawdown water levels, with a high penalty factor associated with violation of these levels. It is represented by the curve shown as a dashed line in the upper half of Figure 1, and it defines the amount of available conservation storage during the irrigation season. Most river basin planning models rely on reservoir operating rules, as defined by the shape of their rule curves. The inherent problem with rule curves is the dependence of their shape on both the storage at the beginning of an irrigation season and the combination of available runoff and water demands throughout the season. To illustrate the problem, imagine there are two back-to-back dry years such that the model cannot reach the full supply level and subsequently cannot follow the prescribed rule curve, which typically assumes a starting position at the full

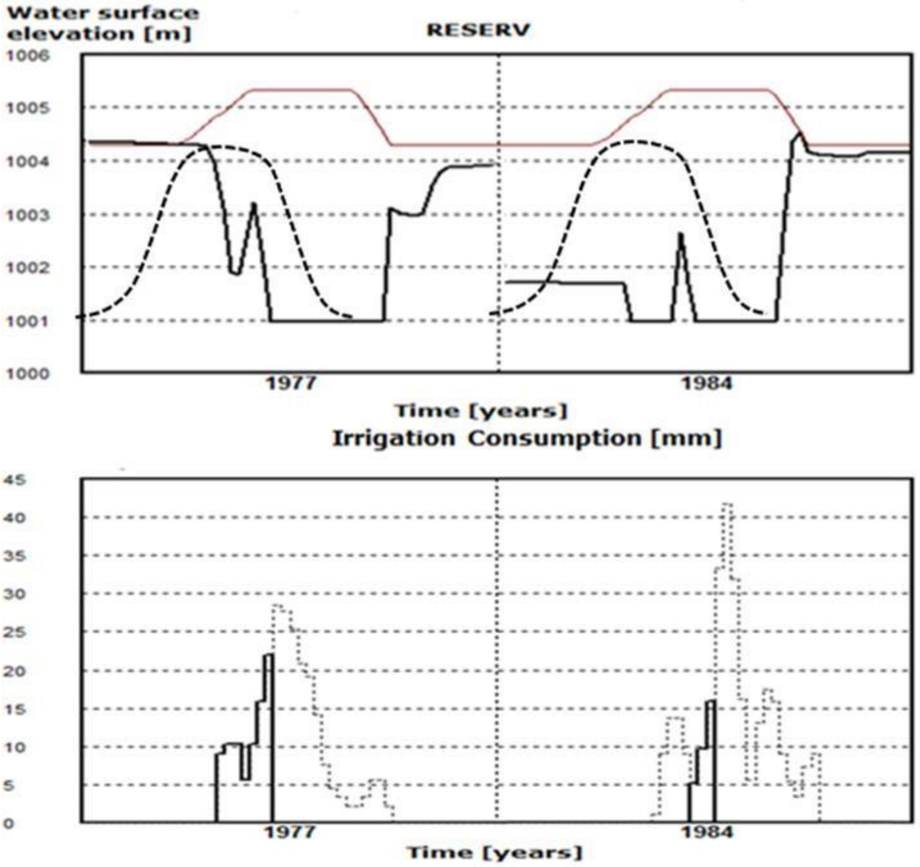

Figure 1. Reservoir levels and achieved vs demanded supply for single time step solutions

supply level. In a strict sense, a pre-defined rule curve represents an attempt to guess the best storage levels that will minimize all water supply deficits for several months in advance. However, the magnitude of water deficits and their distribution should be solved by the model, rather than assumed by the user. If a reservoir rule curve is defined as the set of ideal elevations at the end of each time step that best meets all reservoir operational objectives, it will have a unique shape for each year, and it should be derived by the model, not input by the user. Just as the ideal rule curve shape is different for every year, it is also related to the amount of hedging of water demands, if and when hedging is required. Therefore, the amount of hedging and the shape of the rule curve are set simultaneously. Importantly, Multiple Time Step Optimization (MTO) should be used to determine both the best rule curve shape and the best hedging levels with starting storage levels and seasonal runoff forecasts as the only inputs, as is explained in the following section.

Some researchers have attempted to develop reservoir operating rules based on the historic decisions of the operators. However, this approach overlooks the fact that the operators often make significant errors of judgment. The main advantage of the MTO modelling approach presented below is that rule curves are not required as model input. Rather, they are provided as part of the model solution, and they are unique for every year. With the proposed MTO approach, future research should focus on better seasonal hydrologic runoff forecasts.

MULTIPLE TIME STEP OPTIMIZATION

Multiple Time Step Optimization (MTO) offers significant improvement in the model results, especially when combined with equal deficit sharing constraints, which is a novel approach for optimized demand hedging, which should become standard practice to ensure getting the best possible solutions in each simulated year. Numerous recent studies and publications have examined advantages of the MTO approach, and it was the basis of the California State Water plan (Lund et al., 2003). An MTO model can find the best operating policy for each reservoir in each year, while simultaneously ensuring that all irrigation blocks supported by the same reservoirs have equal deficits throughout the irrigation season. To achieve this, the model should:

a) Optimize over multiple time steps;
b) Use equal deficit sharing constraint within each irrigation season; and,
c) Avoid applying any user defined rule curves, since the model will derive them as part of the optimal solution.

The MTO model solutions constitute perfect rule curves derived uniquely for each hydrologic year. To explain the approach, a simultaneous water allocation optimization over three time steps is illustrated on a simple model schematic in Figure 2 below, which includes one reservoir, two river reaches, one diversion canal and one irrigation block. The same approach can be used to solve the entire hydrologic year for much larger systems. Decision variables are channel flows and storage volumes at the end of each time step, designated as variables $X_{i,t}$ in Figure 2, with subscripts for component *i* and time interval *t*. Variables are assumed to be in units of flow as explained below, so that $X_{2,0} = V_0 / t$ represents the reservoir storage (in the units of flow) at the start of the simulation, t = 0. If the model were set up to run single time step solutions, only the left third of Figure 3 would be shown, consisting of the reservoir in the first week with inflow, outflow channel, initial and ending storage volumes, and diversion $X_{1,1}$ into an irrigation block with its demand of $D_{1,1}$. If the reservoir storages and available inflow provide sufficient water, then $X_{1,1} = D_{1,1}$ (i.e. supply equals demand) and there are no deficits.

The initial storage at the beginning of the first time interval $X_{2,0}$ is given, while variables $X_{2,t}$ represent ending storage at the end of each subsequent time interval t, which becomes the starting storage at the beginning of the next time interval. Net evaporation is omitted in this example, but is modeled in principle as a gain or loss of flow along the reservoir carry-over storage arc. To define an objective function for the model in Figure 2, assume a weekly time interval, a small value of P_i per one unit of storage corresponding to 1 m^3/s of flow over the length of the weekly

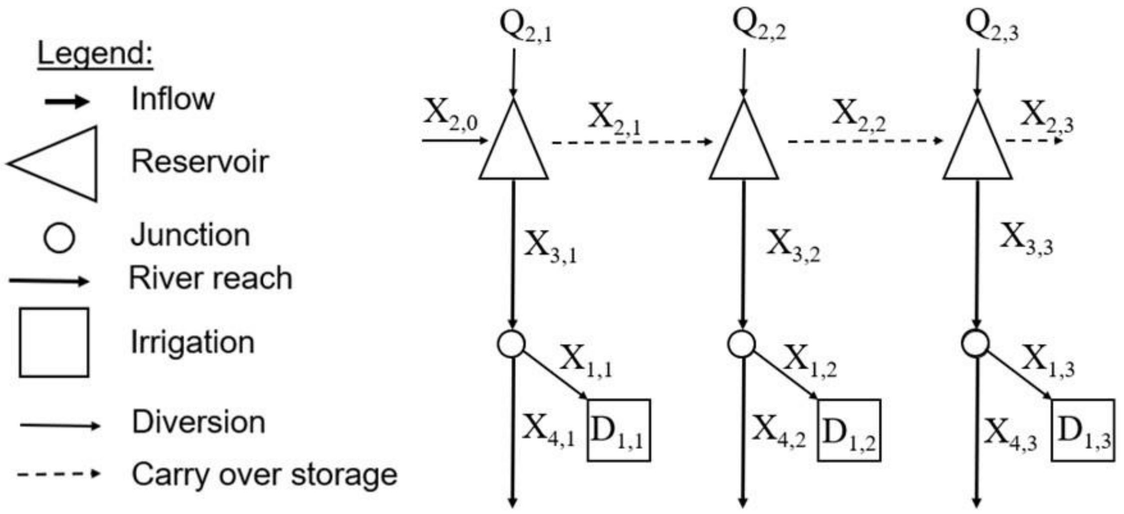

Figure 2. Sample model configuration for solving three time steps simultaneously

time step. Storage also requires some associated value, so that the model avoids spilling water from the reservoir unnecessarily. Assume that supply to irrigation blocks defined by variables $X_{1,1}$, $X_{1,2}$ and $X_{1,3}$ has a higher value for 1m^3/s of supplied flow than storage. All other cost factors associated with flows in the two river reaches (one before and one after the diversion into the irrigation block) can be set to zero in this simplified example (although it is not a general case). The objective function is then specified as

$$Max \sum_{i=1}^{n} \sum_{t=1}^{m} X_{i,t} P_i \tag{1}$$

Where n = 4 is the number of components, since it involves one reservoir, one irrigation block and two river reaches, and m = 3 is the total number of time intervals in Figure 3, although their number is usually set to 52 to cover all weeks within a year. Note that carry over storage also acts as a variable, since it allows the model to balance storage among various time intervals. The above objective function is subject to the following constraints:

$$X_{2,t} + X_{3,t} - X_{2,t-1} = Q_{2,t} \text{ (balance equations for reservoir, } t = 1,m\text{)} \tag{2}$$
$$X_{1,t} - X_{3,t} + X_{4,t} = 0 \text{ (balance equation for irrigation diversion node, } t = 1,m\text{)} \tag{3}$$
$$0 \le X_{i,t} \le U_{i,t} \text{ (lower and upper bounds on all variables, } i = 1,n;\ t = 1,m\text{)} \tag{4}$$

Upper bounds represent limits on storage, canal capacity, or irrigation demand. To ensure even spread of deficits throughout an irrigation season in a dry year, an equal deficit constraint is added that equates the ratio of supplied and demanded quantities in all time intervals:

$$\frac{X_{1,t+1}}{D_{1,t+1}} = \frac{X_{1,t}}{D_{1,t}} \tag{5}$$

NARMADA RIVER BASIN TEST PROBLEM

The test problem for various models presented in this paper was inspired by a request for a model selection made by the Narmada Control Authority (NCA). This test problem was a simplified representation of the Narmada River basin, with the input data and the operational constraints defined on the basis of the NCA's needs that were clearly defined at the outset. The recent tender for Narmada model selection was the first time that the quality of the model solution was used as part of the tender score evaluation, with a total weight of 50% given to model solution. To make the above process feasible, water management objectives in the test problem had to be clearly defined, implying that there is one solution that is the best in terms of its values of the objective function and its ability to satisfy the physical and operational constraints that define the test problem. Detailed problem definition and input data are available on optimal-solutions-ltd.com. The following model features were required in order to solve the test problem posed by the NCA:

- Target flows for irrigation canals in India are set three times a month. Hence, the time step length should be flexible to accommodate durations of 8, 9, 10 or 11 days.
- Narmada River basin has some large reservoirs which have significant net evaporation losses. Hence, accurate modelling of net evaporation is important.
- Ability to find the best possible reservoir operation for any simulated year. Models should be able to solve 36 multiple time steps simultaneously for the entire hydrologic year.
- There are equal deficit sharing policies among some irrigation blocks, both in space (if they share the same source of supply) and in time (implying that deficits, when inevitable, are evenly distributed throughout the hydrologic year)
- Test simulation begins on July 1st of 2008, and ends on June 30th, 2017, lasting 9 years in total. Solutions should be derived assuming a perfect hindsight of runoff and demands for 36 time steps ahead starting on July 1st of each of the 9 years. There are also minimum storage levels that have to be maintained on June 30th of each year. These storage levels were provided by the NCA.
- Maximum flow in a diversion canal may be restricted by the amount of storage during period when storage levels are low. The selected model should have the capability to set the outflow constraint on the diversion canal dynamically as a function of storage.

A simplified version of the Narmada River Basin is represented by a modelling schematic shown in Figure 4. The modelling schematic in Figure 4 shows a list of components, identified by their component numbers that are arbitrarily assigned by the user, and their connectivity. Reservoirs model storage, releases, and net evaporation losses. Reservoir levels should end the hydrologic year on June 30th of each simulated year with a level that is equal or greater than the designated minimum storage levels which were provided by NCA. Bargi reservoir releases are driven by environmental flows in channel 10, and irrigation requirements on blocks 105 and 110. These two irrigation blocks are expected to share deficits evenly in time and in space within each year in all scenarios. During high floods, Bargi storage will be operated to help reduce flows in any of the downstream channels (10 or 15 in this scenario) such that overbank flooding is minimized. Storage levels above the normal water levels are allowed only during time intervals when the full bank capacity is reached or exceeded on either or both of channels 10 or 15. Similar

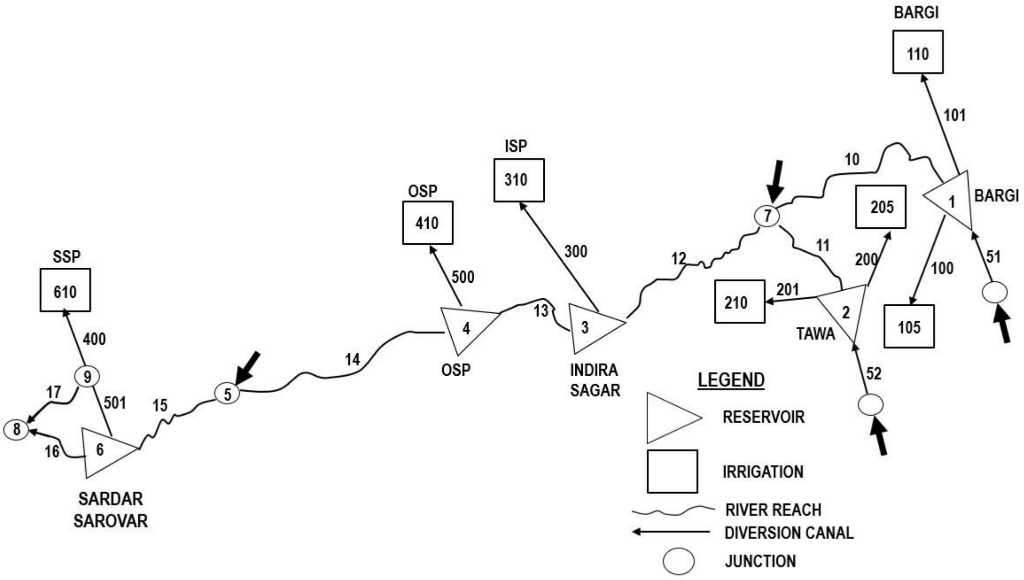

Figure 3 Narmada river basin modeling schematic

operating regime was set up for the Tawa and ISP reservoirs. Input data for the test problem as well as the benchmark solution can be downloaded from www.optimal-solutions-ltd.com.

The tender was posted on the Government of India procurement web site two months. The overall response was rather surprising. Some of the best-known model vendors did not respond, in spite of showing initial interest and asking questions about the tender contents. In addition to the benchmark solution provided by the WEB.BM model which remained unmatched by any vendor, the total of seven model solutions were provided by six vendors. The first surprising finding is that four out of seven models violated the problem constraints, including the mass balance at a node. They are not named explicitly, but their results are shown in Tables 1 and 2.

Table 1. Objective function Values from participating vendors

Tested Model	Objective Function Values			Total objective Function
	Flooding	Environmental Flows	Irrigation Supply	
LP_SOLVE	0	0	134334	134334
GA-MODSIM	0	0	138392	138392
OASIS	31145	8267	140271	179683
Model 1	2037684	198254	124773	2360711
Model 2	4906223	3139	68946	4978308
Model 3	3033479	2458465	930966	6422910
Model 4^{1}	6302746	2419094	210822	8932662
Model 4^{2}	153932450	895026	3015142	157842618

Table 2. Violation of constraints by some of the participating models

Model	Type of Constraints That Were Violated by the Selected Models				
	Min / Max Reservoir Levels	Bargi Right canal flow limits	Net Evap. constraint	Equal Deficit constraint	Mass Balance constraint
Model 1	✓			✓	
Model 2			✓		✓
Model 3		✓		✓	
Model 4^{1}			✓	✓	✓
Model 4^{2}	✓		✓	✓	✓

Model number 4 was used by two independent vendors, options 1 and 2 indicate that their results were different. Several conclusions can be made on the basis of the above results: a) there is a need to start conducting regular audits and additional tests of the existing models for basic constraints such as the mass balance at a node, which have been taken for granted in the past; b) rule curve based models were the worst performers; c) all three top models use liner programming in the solution procedure in some form, and so does the WEB.BM model used as a benchmark; which clearly shows that linear programming outperforms other optimization algorithms; and, d) the top two solutions violated the operating rules by allowing upstream reservoirs to assist the ISP reservoir with water supply during dry years, which was not part of the intended operating policy. Another surprising finding is that there was no vendor that managed to solve the test problem correctly and match the benchmark solution provided by WEB.BM.

DAMODAR RIVER BASIN OPTIMIZATION

Damodar River Basin is located in West Bengal. There are five reservoirs in the basin, designed for multi-purpose water use that includes water supply for irrigation, hydro power generation and flood protection in the downstream reaches, as shown in the modeling schematic in Figure 4. The basin is currently operated using previously developed static reservoir rule curves.

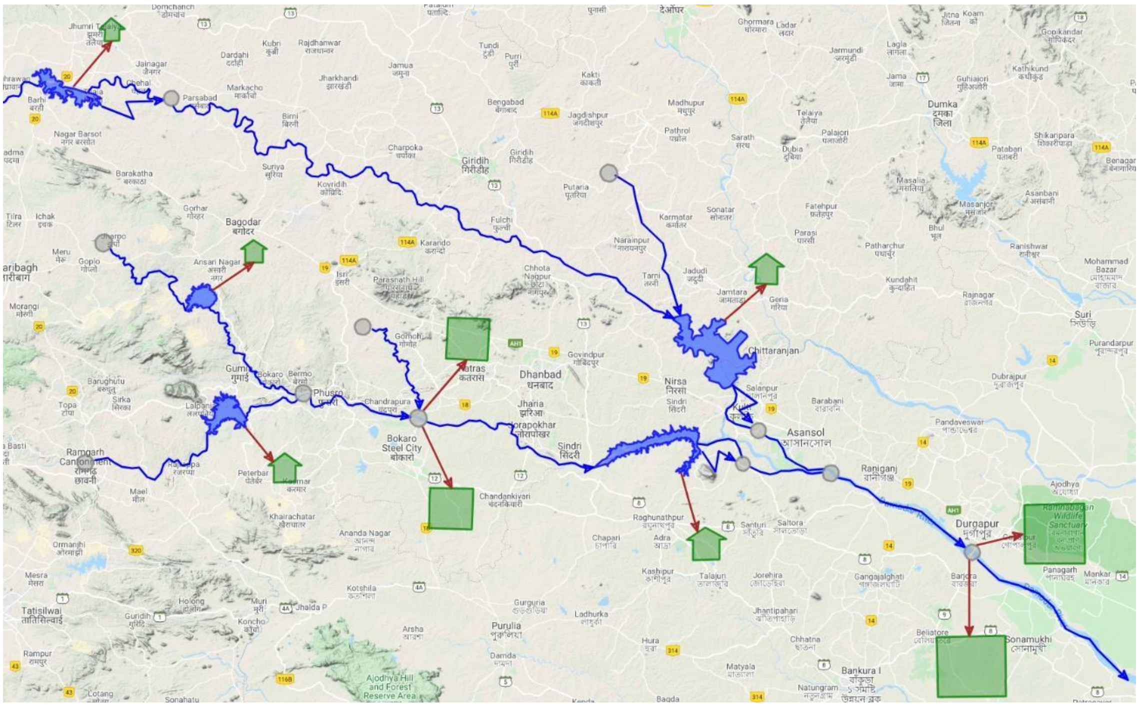

Figure 4. Damodar River Basin Modelling Schematic

Panchet and Maithon are the largest dams in Damodar basin with the total installed hydro power plant capacity of 80 and 60 MW, respectively. The management challenge in this basin is to operate the five reservoirs in the system so as to minimize flood damage, while simultaneously maximizing hydro power production and water supply to irrigation, municipalities and industries. There were 35 years historic hydrologic data provided in this study, starting in 1981. A continuous simulation was set up with a variable time step length, with 10 daily averages in dry season, and 3-day averages in the monsoon season. The following scenarios were set up:

a) Verification scenario: historic reservoir outflows in this scenario were enforced and the resulting simulated reservoir levels were compared with the historic levels. A good match between the historic and simulated reservoir levels confirms correct runoff estimates.
b) Optimization scenario: the same runoff estimates confirmed in the verification scenario were used, and the same starting reservoir elevations in 1981, but the model was set up to find the best reservoir operation that maximizes the stated objectives.
c) 6-day runoff forecast scenario, which utilizes alternative operating rules designed on the basis of the results under b) and the assumed availability of 6-days of runoff forecast, which implied MTO solutions for two time steps simultaneously (each time step assumes average flows over 3 days), with the solution for the first time step adopted as final before moving to the subsequent time step.

To avoid downstream flooding, the model was set up to keep the combined outflows from Maithon and Panchet to less than 3300 m^3/s. Historic operation did not necessarily meet all target demands stated by the DVC corporation, although the difference is relatively small with respect to the target. It should be noted that much of the diverted water returns to the stream, but at the time this study was conducted, return flow quantities were not available. Both historic and optimal scenarios included net evaporation losses on reservoirs as a function of the average reservoir area over a time step. Historic operation showed water levels in the flood storage zone (above 146.3 m for Maithon and 124.97 m for Panchet) in many years in which there was no flooding. It is not clear why this was allowed. Also, in many historic years reservoirs are drawn down much more than necessary, probably due to the policy to follow a similar rule curve in each year, which causes unnecessary loss of generated power. As shown in Table 3, optimized reservoir operation shows improvements compared to the historic run for all types of water use, and completely eliminates flood damage.

Optimal solutions shown in Table 3 are based on perfect foreknowledge of inflows for the entire year. The obvious question is now “how do we use this information?”, especially since the historic series of 35 years will never repeat exactly the way they unfolded since 1981. One of the ways to answer this question is to resort to the use of stochastic hydrology (Ilich, 2013), which can provide a much longer (1000 years) of statistically similar hypothetical runoff series of local inflows into each reservoir, solve all hypothetical 1000 years with an optimization program, and obtain 1000 perfect responses (rule curves) that match various starting levels and inflow conditions, which can serve as a learning database for development of better reservoir operating rules. Several options have been investigated to this end, and the one that proved the most successful is based on reshaping the reservoir operating zones based on statistical analyses of 1000 optimal solutions, which implies that the model has produced 1000 simulated water levels for the end of each time step. Those levels were all developed using optimization. They can then be analyzed such that a probability density function is produced for the end of each time step. The next step is to connect the elevations with the same probability of exceedance at the end of each time step as shown in the bottom half of Figure 5 that shows four operating zones (maximum level to 20 percentile, 20-50 percentile, 50-80 percentile, and 80 percentile to the top of dead storage zone. If the zones are labelled from 1 to 5, the model will attempt to keep all reservoirs in the same respective operating zone. For example, the model will prevent a situation where one reservoir is in zone 5 while the others are in zone 2. Reservoir operating zones designed in this way provide operating rules for low flow season that are easier to follow, based on the demand driven releases and the sequence of releases from each reservoir based on maintain all reservoirs within the zone of the same order. These rules will eventually determine the starting storage levels at the beginning of a flood event during the monsoon season, where the calculation time step is much shorter (3 days in this study), and the MTO simulation is run consecutively for two time steps simultaneously over a 6 day flow forecast period. In other words, a 3-day warning period is

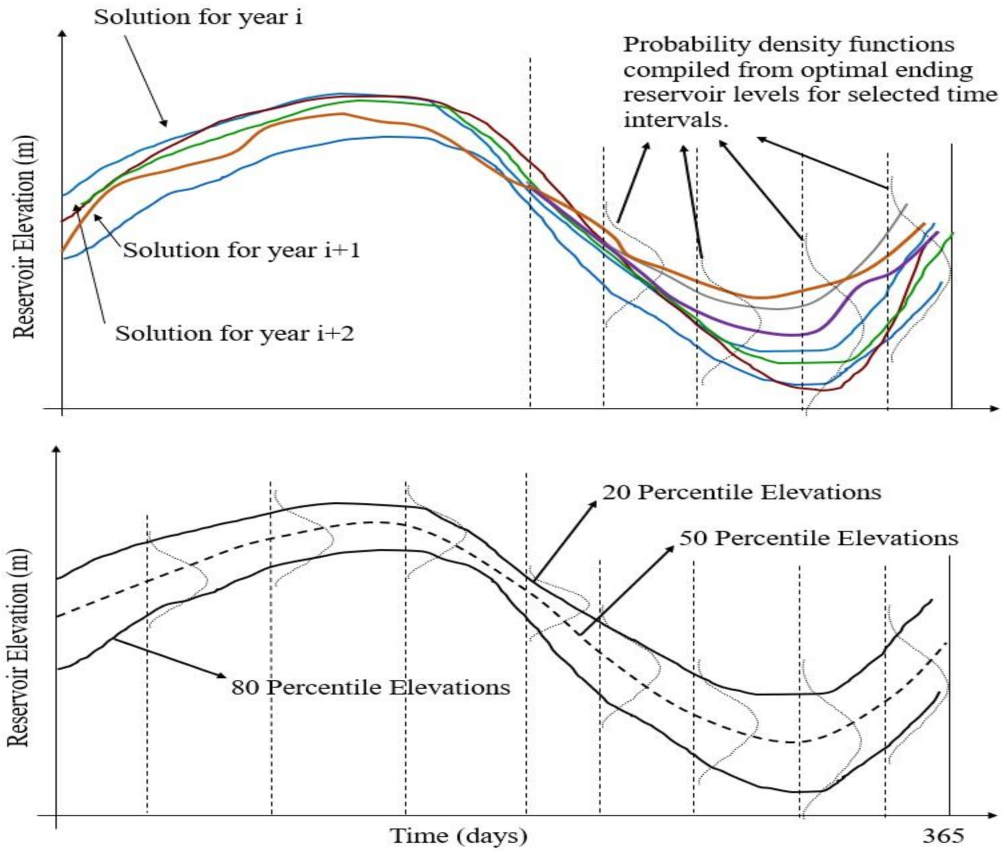

Figure 5. Use of optimal solutions in shaping reservoir operating zones

assumed for an excessive incoming flood, thus giving the model and opportunity to adjust increase the storage releases accordingly before the flood peak reaches the Maithon and Panchet reservoirs. This increases the flood control storage on these reservoirs, thus helping alleviate the flood. In the same time, the built-in optimization program coordinates the operation of all five reservoirs to ensure that the downstream full bank flow capacity is not exceeded. The final results of this scenario are shown in Table 3, along with the verification run (historic operation) and the optimization run with a perfect hindsight for each hydrologic year.

Table 3. Summary comparison of historic and optimal basin operation (mean annual output)

Modeling Scenario	Tilaiya (GWh)	Maithon (GWh)	Panchet (GWh)	Total Water Use (10^6 m^3)	No. of years* with floods (Q > 3300 m^3/s below Durgapur Barrage)
Historic Verification	10.49	180.62	205.29	4751	6
Optimal MTO	12.86	300.89	393.23	5222	0
6 days flow forecast	10.18	276.37	358.23	5100	0
Installed Power (MW)	4	2x60	2x80	-	

*Number of years out of the 35 simulated years, hence flood occurrence is 6/35 = 17% of the time

The overall improvements of the 6-day flow forecast scenario shown in Table 3 are not as good as the improvements based on the perfect hindsight of runoff, and that is to be expected. Still, the data in Table 3 can be read as follows: given the adherence to the (a) new reservoir operating rules developed on the basis of the optimal solutions in the “Optimal MTO” scenario; (b) the availability of reliable 6 day runoff forecasts; and, (c) the repeated use of MTO optimization within the 6 days time horizon to drive the real time operation, the basin managers would have seen the following improvements over the bast 35 years: (a) drastic reduction of flood damages to zero for all 6 historic floods in this period; (b) increased water supply by around 350 million m^3 per year on average (5100 – 4751 in Table 3); and (c) increased generated power by 62% on average per year. It would appear that investing in better data monitoring and water management tools may be a worthy investment for the Government of India, which was the basic idea behind forming the National Hydrology Project.

CONCLUSIONS AND RECOMMENDATIONS

River basin modeling in India has so far been based on the use of modeling of user supplied reservoir rule curves. This approach delivers inferior solutions compared to optimization-based models. Also, users should begin to pay attention to results of some of the existing models as they may be violating important constraints. Optimization models should first be used to help develop river basin plans, before attempting to apply them as seasonal operational tools. A good plan is based on comprehensive effort to develop historic time series and provide all other background hydrologic and water demand analyses, in addition to analyzing the optimization model results. Finally, a combination of optimization, stochastic hydrology and improved runoff forecasting capabilities holds out the promise of developing sophisticated water management tools that may aid future real time reservoir management in a substantial way.

REFERENCES

Colorado State University, 2020. Modsim DSS: http://modsim.engr.colostate.edu/

Danish Hydraulic Institute, (2019). Mike 11 model. https://www.mikepoweredbydhi.com/ Hydrologic Engineering Center, US Army Corps of Engineers (2019). HEC-RAS. https://www.hec.usace.army.mil/software/hec-ras/

Government of Victoria, Australia, 2020. *REALM - Resource Allocation Model*, https://www.water.vic.gov.au/water-reporting/surface-water-modelling/resource-allocationmodel-realm

Haro D., J. Paredes, A. Solera, J. Andreu 2012. A Model for Solving the Optimal Water Allocation Problem in River Basins with Network Flow. *Water Resources Management,* 26, 4059-4071. DOI 10.1007/s11269-012-0129-7

Hydrologic Engineering Centre. 2020. HEC-ResPRM User Manual. *US Corps of Engineers*. https://www.hec.usace.army.mil/software/hec-resprm/

Hydrologic Engineering Centre. 2019. HEC-ResSIM software and User Manual. https://www.hec.usace.army.mil/software/hec-ressim

Ilich, N. 2013. An Effective Three-Step Algorithm for Multi-Site Generation of Weekly Stochastic Hydrologic Time Series. *Hydrological Sciences Journal,* 59 (1 – 2) 2014. http:/dx/doi.org/10.1080/02626667.2013.822643

Ilich, N. 2008. Shortcomings of Linear Programming in Optimizing River Basin Allocation. *Water Resources Research*, Vol. 44, W02426, doi:10.1029/2007WR006192.

Ilich N. 2009. Limitations of Network Flow Algorithms in River Basin Modelling. *ASCE Journal of Water Resources Planning and Management*, Vol. 135 (1) doi: 10.1061/(ASCE)0733-9496 135:1(48).

Ilich, N. 2020. WEB.BM–Web Basin Management Model. www.riverbasinmanagement.com

Ilich, N. M. Sinha and N. Manglik. 2019. Innovative Approach Towards Model Selection for Narmada River Basin Modelling. *2nd International Conference on Sustainable Water Management,* 6 – 8 November 2019, Pune, Maharashtra, India. http://nhp.mowr.gov.in/Conference-2/Home.aspx

Labadie, J.W. (2004). Optimal Operation of Multi-reservoir Systems: A State-of-the-Art Review. *Journal of Water Resources Planning and Management,* 130 (2), p. 93-111.

Lund, J. et al. Statewide Economic-Engineering Water Model – CALVIN. *University of Calif. - Davis.* 2003. http://www.waterplan.water.ca.gov/docs/tools/descriptions/CALVINdescription.pdf

Needham, J.T., D. W. Watkins Jr., and J. Lund. (2000). Linear Programming for Flood Control in the Iowa and Des Moines Rivers. *Journal of Water Resources Planning and Management,* 126 (3), p. 118-127.

Maier, H.R ; Kapelan, Z ; Kasprzyk, J ; Kollat, J ; Matott, L.S ; Cunha, M.C ; Dandy, G.C ; Gibbs, M.S ; Keedwell, E ; Marchi, A ; Ostfeld, A ; Savic, D ; Solomatine, D.P ; Vrugt, J.A ; Zecchin, A.C ; Minsker, B.S ; Barbour, E.J ; Kuczera, G ; Pasha, F ; Castelletti, A. (2014). Evolutionary algorithms and other metaheuristics in water resources: Current status, research challenges and future directions. *Envir. Modelling and Software,* Vol.62, pp.271-299.

Randall, D., L. Cleland, C. S. Kuehne, G. W. Link and D. P. Sheer. (1997). Water supply planning simulation model using mixed-integer linear programming 'engine.' *Journal of Water Resources Planning and Management,* 123 (2), p. 116–124.

Ravelle, C, and W. Kirby. (1970). Linear Decision Rule in Reservoir Management and Design: 2. Performance Optimization. Water Resources Research, 6 (4), p. 1033-1044.

Wurbs, R. (1993). Reservoir System Simulation and Optimization Models, *Journal of Water Resources Planning and Management,* 119 (4), 455-471.

Zagona, E.A., T.J. Fulp, R. Shane, T. Magee and H.M. Goranflo. (2001). RiverWare: A generalized tool for complex reservoir system modelling. *Journal of the American Water Resources Association*. 37 (4), p. 913-929.